Solid state physics 2018 Final Exam

20th April 2018

Good luck!

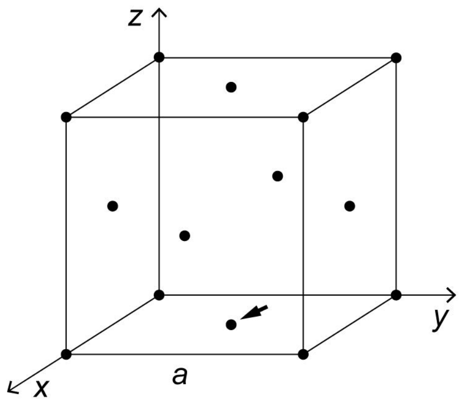
You may not use textbooks, notes, or calculators.

Fill out your student number, your name, and your family name below. Only provide answers inside the answer boxes.

1. **Crystal structure**

   (a) Write down definitions of: a primitive unit cell, a conventional unit cell, and a Wigner-Seitz unit cell. *(5 points)*

(b) Consider the FCC lattice shown below with lattice constant a. Write down the basis using the conventional unit cell. Compute the structure factor and determine the selection rules of (h, k, l) which give both constructive and destructive interference. *(5 points)*



(c) **Distorting Crystal structure**

We now displace the atom marked by an arrow in the image above in the positive z-direction by a distance δ as well as all the atoms equivalent to that one by the translations of the conventional unit cell.

Write the *primitive* lattice vectors for the cases $\delta = 0$ and $0 < \delta < a$. *(10 points)*

(d) Compute the structure factor for the scattering vector (h, k, l) as a function of δ. Then use this expression to determine the scattering amplitudes for scattering vectors $(1, 1, 1)$, $(1, 1, 0)$ and $(0, 0, 1)$. *(10 points)*

(e) Which additional diffraction peaks appear when $\delta \neq 0$, compared to $\delta = 0$? Which peaks stay absent when $\delta \neq 0$? *(10 points)*

(f) Compute the intensity of the new peaks that appear for $\delta > 0$ when $l \ll a/\delta$. Explain how the intensities of these diffraction peaks may be used to determine δ in an X-ray experiment. *(10 points)*

2. **Magnetic semiconductor**
An undoped semiconductor with electron and hole effective masses m_e and m_h is placed in an external magnetic field, so that all electrons experience a Zeeman splitting $g\mu_B B$. This makes the dispersion relation of electrons with spin up differ from that of electrons with spin down (in both the conduction and the valence band). We consider a sufficiently weak magnetic field, so that $g\mu_B B \ll E_G$, with E_G the semiconductor band gap. *In this problem if you do not know how to deal with Zeeman splitting, disregard spin-related phenomena to get partial points in any subquestion except for the last one.*

(a) Draw the band structure of such a semiconductor, showing both the bands of spin up electrons and spin down electrons. *Hint: check that when $B = 0$ your answer coincides with band structure of a semiconductor. (10 points)*

(b) Draw the density of states $g(E)$ of this semiconductor and indicate the relevant energy scales. *Hint: once again check that when $B = 0$ your answer coincides with density of states of a regular semiconductor. (10 points)*

(c) Compute the number of electrons and holes with spin up and spin down as a function of E_F. Write down the charge neutrality condition for this material. Use that you may approximate Fermi-Dirac distribution as Boltzmann distribution when $E_G \gg k_B T$. *Hint: once again your answer should agree with a regular semiconductor when* $B = 0$. *(10 points)*

(d) Using the charge neutrality condition determine the concentrations of electrons and holes with spin up and spin down when magnetic field is large $g\mu_B B \gg k_B T$. **Hint: two of the four concentrations will be small.** *(10 points)*

(e) Using the answer of the previous subproblem, compute the total magnetization M of this semiconductor due to spins when $g\mu_B B \gg k_b T$. Sketch $M(B)$, the dependence of magnetization on B. *(10 points)*

3. **Extra answer space**