Solid state physics (TN2844) 2025
Final exam (180 minutes)

May 20, 2025

Exam details:

- If you need extra answer space, use the extra spaces provided on the last page and indicate in the corresponding question that you have done so. If you still need more space: Ask for an extra exam copy, fill in your name and continue writing the solution.
- You may use only the provided formula sheet. You may not use the course book, lecture notes, or other materials.
- You may not use calculators or other digital tools.
- This exam contains 4 problems on 12 pages (including the cover).

Page:	2	3	4	5	6	7	8	9	10	Total
Points:	15	25	20	20	20	10	10	20	10	150

Good luck!

TU Delft Delft University of Technology

This exam is designed and reviewed by A. Akhmerov and T. van der Sar

1. **1D continuum model with spin-orbit coupling and magnetic field**

   We consider a 1D Hamiltonian:

$$H(k) = \begin{pmatrix} \frac{\hbar^2}{2m}(k+k_0)^2 & B \\ B & \frac{\hbar^2}{2m}(k-k_0)^2 \end{pmatrix} \equiv \frac{\hbar^2}{2m}(k^2+k_0^2)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \hbar^2 k_0 k/m & B \\ B & -\hbar^2 k_0 k/m \end{pmatrix},$$

   where the two states are the spin-up and spin-down states of electrons in the system. Here $k_0 > 0$ is a momentum shift, B is the spin coupling due to the magnetic field, k is the 1D wave vector, and m is the effective mass. Two equivalent ways of writing this Hamiltonian are presented above for your convenience.

   (a) (5 points) Compute the dispersion relation of this system.

   (b) (10 points) Compute the effective mass m^* at finite B for both bands at $k = 0$. Hint: first expand the dispersion relation around $k = 0$.

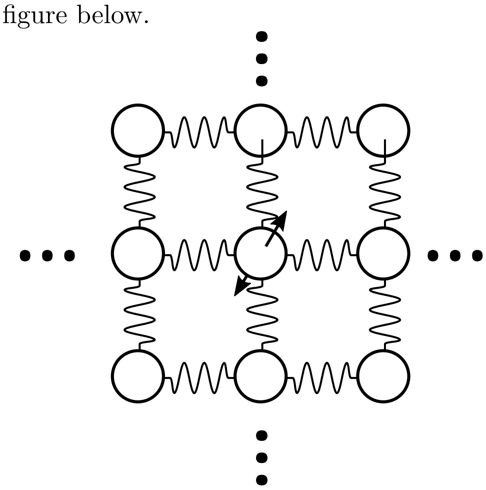
(c) (15 points) Consider $B = 0$, when the Hamiltonian is diagonal. Hint: your answer from part (a) is not convenient for this question.

- Sketch the dispersion relation.
- Based on your sketch, sketch the density of states $g(E)$.
- Compute the expression for $g(E)$. Hint: consider if the density of states depends on k_0.
- Sketch the density of states for the lower band $g_-(E)$.

(d) (10 points) Determine how the electron heat capacity scales with temperature and with Fermi energy at $B = 0$ when $T \to 0$.

2. **Vibrations of a membrane**

We consider a square lattice of atoms that are connected by springs and move in the direction z perpendicular to the plane, as shown in the figure below.



The lattice constant is a, the mass of each atom is m, and the returning force acting on the atom with displacement u_1 and connected to another atom with displacement u_2 is equal to $\kappa(u_1 - u_2)$.

(a) (10 points) Write down the equations of motion of the atom in the unit cell with indices n, m (i. e. with displacement $u_{n,m}$).

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(b) (10 points) Write down the appropriate plane wave ansatz, substitute it into the equations of motion, and compute the dispersion relation. Check that if $k_y = 0$ (where k_y is the y-component of the wave vector) you reproduce the dispersion relation of an atomic chain.

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(c) (10 points)

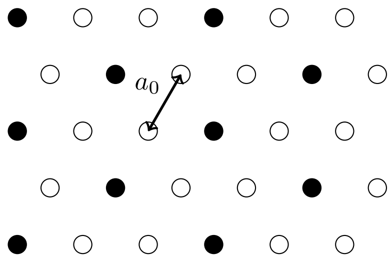
- What is the sound velocity in this membrane?
- Compute the density of states at $\omega \ll \sqrt{\kappa/m}$.

(d) (10 points)

- If the lattice constant in x-direction was changed to $2a$, argue what would happen to the sound velocity in x- and y-direction?
- If the spring constant κ in x-direction was doubled, argue what would happen to the sound velocity in x- and y-direction?

3. **Crystal structure**

Consider the following 2D crystal structure with two types of atoms and the distance a_0 between all neighboring atoms.



(a) (10 points) Draw primitive lattice vectors on the crystal structure above. Then determine the distance between the lattice points and express the primitive lattice vectors in Cartesian coordinates. Finally, write down the basis of the unit cell.

(b) (10 points)

- Compute the reciprocal lattice vectors of this crystal structure.
- Calculate the structure factor for this lattice, assuming that the black atoms have structure factor f_A and the white atoms have structure factor f_B.

(c) (10 points) Determine if any diffraction peaks are absent when $f_A \neq f_B$ and when $f_A = f_B$.

4. **Two-valence band 2D semiconductor**

Consider a strictly **two-dimensional** semiconductor with a conduction band and two valence bands. The Hamiltonian near the band extrema can be written as $H_c(\vec{k}) = E_C + \frac{\hbar^2 k^2}{2m_c}$ for the conduction band, and $H_{v1}(\vec{k}) = E_{V,1} - \frac{\hbar^2 k^2}{2m_1}$, $H_{v2}(\vec{k}) = E_{V,2} - \frac{\hbar^2 k^2}{2m_2}$ for the two valence bands. The energy separation between the valence bands is $\Delta = E_{V,1} - E_{V,2} > 0$, and the energy gap $E_G = E_C - E_{V,1} \gg k_B T$. The effective masses satisfy $m_1 > m_2$.

(a) (10 points) Calculate and sketch the density of states $g(E)$ of this semiconductor. Clearly indicate the contribution from each band and the relevant energy scales on your sketch.

(b) (10 points) Calculate the concentration of electrons in the conduction band and holes in each of the valence bands as a function of the Fermi energy E_F, temperature T, and the respective effective masses. Assume the Fermi level is in the band gap far from all bands and the Boltzmann approximation is valid.

(c) (10 points) First, derive the charge neutrality condition for this semiconductor. Then, solve for the equilibrium Fermi energy E_F in the intrinsic case when $\Delta \gg k_B T$. Finally, determine the concentrations of electrons and holes in each band.

(d) (10 points) Sketch the optical absorption spectrum of this semiconductor. In your sketch indicate the relevant energy scales.

Extra answer space

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