Solid state physics 2020 Minitest 1 (120 minutes)

23 February 2021

Good luck!

1. (50 points) **Anisotropy in a three-dimensional bosonic dispersion**

   We consider bosons in three dimensions with an anisotropic dispersion relation $\omega = \gamma_1(k_x^2 + k_y^2) + \gamma_2 k_z^2$, where $\gamma_1 \neq \gamma_2$ and only a single polarization (such particles exist and they are magnetic excitations in some materials—magnons).

   (a) (5 points) Explain in words the concepts of dispersion relation and density of states.

   (b) (15 points) Compute the density of states of these bosons. Check that your resulting $g(\omega)$ scales with $\sqrt{\omega}$ (if you are unable to solve this question, use $g(\omega) = C\sqrt{\omega}$ in the next subquestions). *Hint: to find the density of states, one needs to restore the spherical symmetry in the integral with a wavevector substitution.*

   (c) (10 points) Compute the heat capacity in the low temperature limit. Leave the definite integral unevaluated as long as it does not depend on any parameters like γ_1, γ_2, or β.

   (d) (10 points) Assuming there is a total number N of these bosonic modes, compute the Debye cutoff frequency ω_D.

   (e) (10 points) Determine the heat capacity in the high temperature limit. What temperature can be considered high in this case?

2. (50 points) **Electron and hole-conductivity in the Drude model**

   We consider a semiconductor, which is a material that can host two types of charge carriers: On the one hand there are electrons with charge $-e$ and mass m_e. On the other hand there are holes: particles with charge $+e$ and mass m_h. The concentrations of the electrons and holes are both equal to n.

   (a) (10 points) Write down the equations of motion (one for the electrons and one for the holes) describing the average acceleration of the electrons and holes in an electric field E. Include a damping force as in the Drude model.

   (b) (10 points) From the equations of motion, derive an expression for the total electrical conductivity $\sigma = 1/\rho$ as a function of n, m_e, and m_h.

   (c) (10 points) We will now derive how we expect the conductivity of this semiconductor to scale with temperature. To do so, we first analyze the effect of temperature on the scattering time τ.

   Assuming that the scattering rate $1/\tau$ is linearly proportional to the number of thermally excited phonons in the system, argue how you expect τ to scale with temperature T when $T \gg T_D$. (Hint: recollect how the number of phonons occupying one mode behaves at high temperatures.)

   (d) (10 points) The concentrations of free electrons and holes in a semiconductor are not constant like in metals, but rather depend on temperature T: $n \propto e^{-E_g/k_BT}$, where $E_g \gg k_BT$ is a constant. How does the total conductivity of this semiconductor scale with temperature? Argue whether the change in τ or that in n dominates.